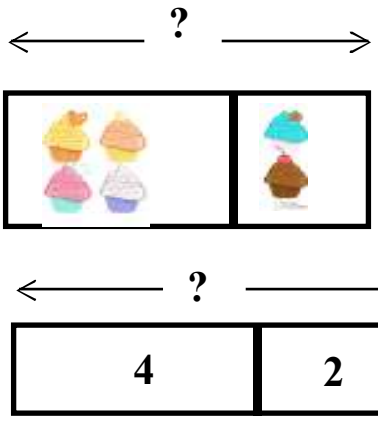
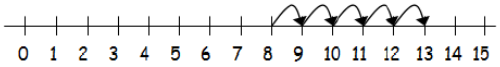
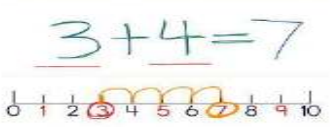
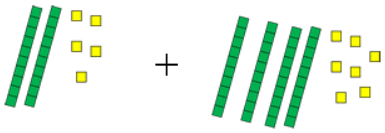
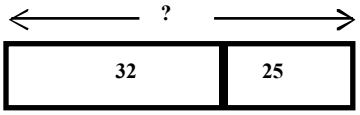
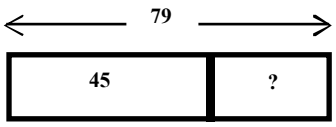
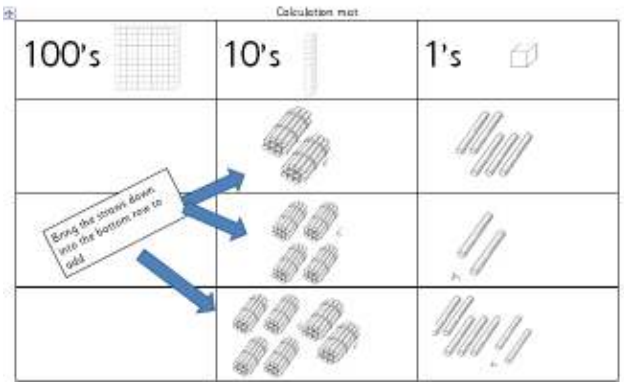
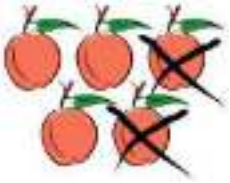
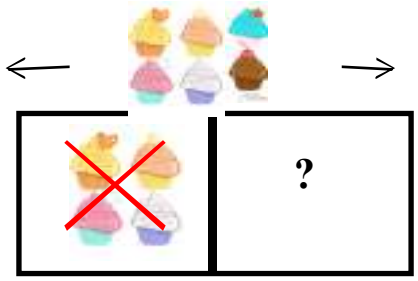
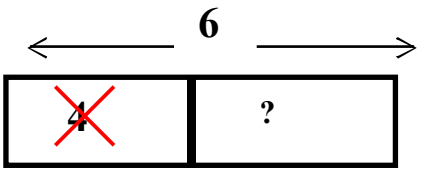
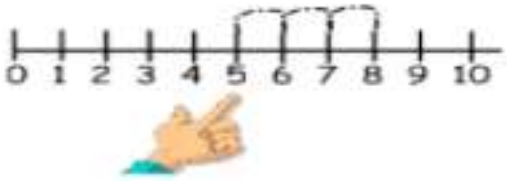
Written methods of calculation for Addition.

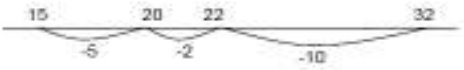
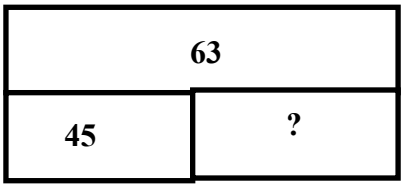
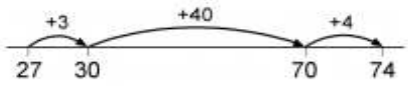
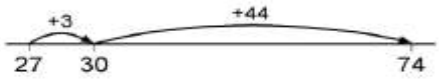
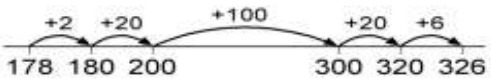
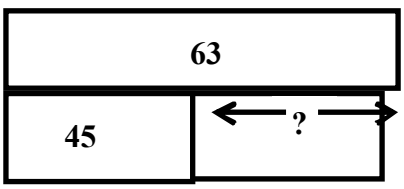
Step 1	
Concrete addition *This step requires the children to combine two groups of objects or images.* *Eg. Count out 3, add two more. How many do we have now?* *Use of fingers is encouraged as this is a constantly available resource.* *The number sentence should be related to the objects/pictures/fingers whenever possible. Eg. 3 teddies + 2 teddies = 5 teddies* *Children need to be able to verbalise calculations in 'everyday language' before introducing symbols e.g. 'I had 3 teddies and you gave me 2 more. Now I've got 5'* *When discussing problems, this should also be introduced as a* **bar model**, *initially drawn round the objects and then blank.*	For example: 3 + 2 = 5  For example  5 + 1 = 6 
Step 2	
Counting on *This step requires the children to realise that there is no need to count from the start each time. They combine a hidden number with a concrete number ie. The first number in their head, the second on their fingers, or 6 cakes already in a box, if I add 2 more, how many do I have now?* *They should be encouraged to discover that it is easier to put the biggest number in their heads.* *This is to be recorded on a* **numbered number line**. *When discussing problems, this should also be introduced with the use of a* **bar model** *as above.*	For example: 5 + 4 = 9  **6 + 2 = ?** 8 + 5 = 13  

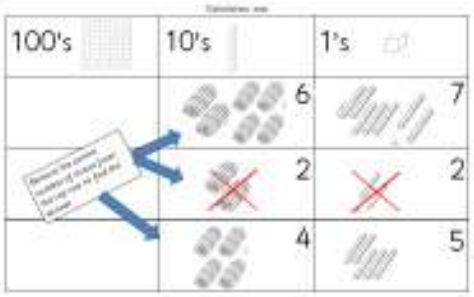
Step 3	
Empty number line *This step requires the children to record addition on a blank number line.* *The steps should be extended so that they bridge through a multiple of ten.* *Additions should involve single digit numbers as well as two and three digit numbers. The method can also be used for adding decimals.* *Children should be encouraged to use their knowledge of number bonds here e.g. if I'm adding 17 + 8 and I know the pairs that make 8 and I know what to add to 17 to get to the next ten, I would probably think of 17 + 8 as 17 + 3 + 5* **Use of straws, Base 10 or Place Value counters should be used to reinforce the value of each digit.** *Problem solving should include the use of* **bar modelling** *to aid visualisation.*	*For example:* 48 + 36 = 84  or   25 + 47 = 72 32 + 25 = 57  45 + ? = 79 
Step 4	
Partitioning *This step requires the children to partition the number, add these partitions and then add the partial sums.* *This method can be used for 3 digit numbers and decimals.* **Use of straws, Base 10 or Place Value counters should be used to reinforce the value of each digit.** *Problem solving should include the use of* **bar modelling** *to aid visualisation as above.*	*For example:* 47 + 76 40 + 70 = 110 7 + 6 = 13 110 + 13 - 123 –––––––––– 324 + 241 300 + 200 = 500 20 + 40 = 60 4 + 1 = 5 500 + 60 + 5 = 561 –––––––––– 45.3 + 56 .8 40 + 50 = 90 5 + 6 = 11 .3 + .8 = 1.1 90 + 11 + 1.1 = 102.1

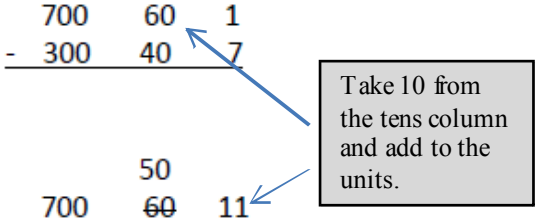
Step 5	
Extended column *This step requires the children to set the calculation out in a column (being careful to ensure correct place value). They are then required to add the* **lowest value digit first**, *recording the answer below before moving to the other digits and adding the partial sums.* *This method can be used when adding 2, 3 or 4 digit numbers as well as decimals.* **Use of straws, Base 10 or Place Value counters should be used to reinforce the value of each digit.** *These should be placed onto a grid clearly marked with Hundreds, Tens, Units (as appropriate)* *Problem solving should continue to include the use of* **bar modelling** *to aid visualisation as in previous steps.*	For example: 83 + 42 5 120 125 367 +185 12 140 400 552 14.28 +17.56 0.14 0.70 11.00 20.00 31.84 *Example using straws on a calculation mat:* 
Step 6	
Short method *This method requires the children to set the calculation out in a column (being careful to ensure correct place value).* *When adding, the children are required to* **begin with the units**, *and carry using correct language such as 'carry ten' or 'carry one hundred'. The number carried should be recorded* **below** *the line.* *This method should be extended to addition of 3, 4 and 5 digit numbers as well as decimals, and can be extended to adding more than two numbers.* **Use of straws, Base 10 or Place Value counters can continue to be used to reinforce the value of each digit.** *Placed onto a grid clearly marked as appropriate.* *Problem solving should continue to include the use of* **bar modelling** *to aid visualisation as in previous steps.*	For example: 367 +185 552 1 1 3587 + 675 4262 1 1 1 72.8 +54.6 127.4 1 1 13.86 + 9.481 23.341 1 1 1

Written methods of calculation for Subtraction.

Step 1	
Concrete subtraction *This step requires the children to physically take away one or more objects from a set of objects.* *Children will also cross out images to take away.* *Use of fingers is encouraged as this is a constantly available resource.* *The number sentence should be related to the objects/pictures/fingers whenever possible. Eg. 5 teddies - 2 teddies = 3 teddies* *When discussing problems, this should also be introduced as a* **bar model**, *initially drawn round the objects, then blank.*	*For example:*  5 – 2 = 3   6 – 4 = ?  
Step 2	
Counting back on a numbered number line *This step requires the children to use a numbered number line to work out one less or several less than a given number.* *When discussing problems, this should also be introduced with the use of a* **bar model** *as above.*	*For example:* 4 – 1 = 3  8 – 3 = 5 

Step 3	
Counting back on a blank number line *This step requires the children to count backwards using a blank number line.* *The number of jumps can be reduced as the children become more proficient.* *They should also be encouraged to use their understanding of the number system to speed up calculations eg. 74 - 27 might be usefully calculated as 74 – 30 + 3* *Problem solving should include the use of* **bar modelling** *to aid visualisation.*	*For example:* 32 - 17 = 15  74 - 27 = 47  63 – 45 = ? 
Step 4	
Counting on using a blank number line *This step introduces the idea of finding the difference and requires the children to count up on a blank number line.* *The number of jumps can be reduced as the children become more proficient. This is especially needed when moving on to two or three-digit numbers.* *Children should be taught to use a combination of Steps 3 & 4 depending on the question. Eg to calculate 168 – 27 it makes sense to count backwards but if I was calculating 168 – 149 I'm more likely to count forwards (find the difference)* *Problem solving should continue to include the use of* **bar modelling** *to aid visualisation.*	For example: 74 - 27 = 47   326 - 178 = 148   *What is the difference between 63 and 45?* *How many more is 63 from 45?* *How many less is 45 from 63?* 

Step 5	
Expanded column method *This step requires the children to partition the numbers and then subtract the* **lowest value digits first**. *It should be used to introduce the idea of decomposition and* **carrying**. *This method should be used for two and three-digit numbers and can be extended to decimals.* **Use of straws, Base 10 or Place Value counters should be used to reinforce the value of each digit.** *These should be placed onto a grid clearly marked with Hundreds, Tens, Units (as appropriate)* *Problem solving should continue to include the use of* **bar modelling** *to aid visualisation as in previous steps.*	For example: 77 - 24 = 53 70 7 - 20 4 50 3 Using straws:  761 - 347 = 414 700 60 1 - 300 40 7 Take 10 from the tens column and add to the units. 50 700 ~~60~~ 11 - 300 40 7 400 10 4



Step 6

Short method (decomposition)

This step requires the children to set the calculation out in a column (being careful to ensure correct place value).

They should subtract the right hand column ***(units) first*** *and carry from the left hand side column if needed.*

Use of straws, Base 10 or Place Value counters should be used to reinforce the value of each digit as before.

This method can be used for any number of digits as well as decimals.

Problem solving should continue to include the use of ***bar modelling*** *to aid visualisation as in previous steps.*

For example:

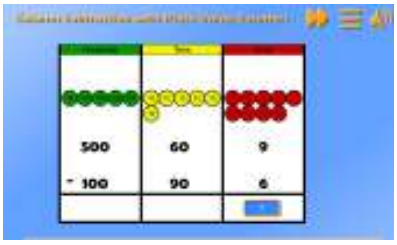
537 - 214 = 323

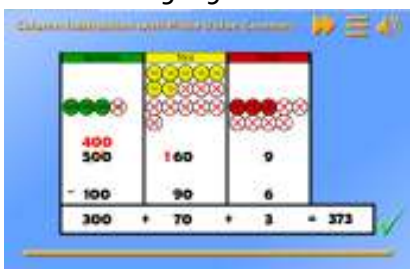
$$\begin{array}{r} 537 \\ -\ 214 \\ \hline 323 \end{array}$$

728 - 51 = 677

$$\begin{array}{r} {}^{6}\not{7}\not{2}8 \\ -\ 51 \\ \hline 677 \end{array}$$

Use of place value counters when carrying:





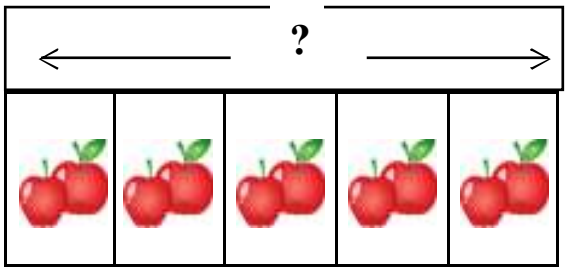
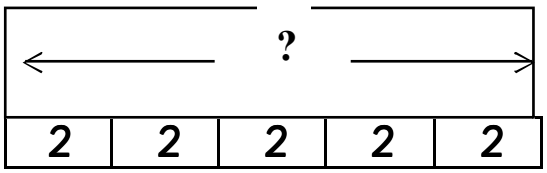
Written method:

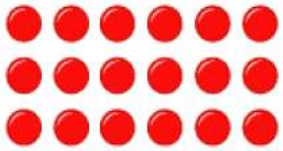
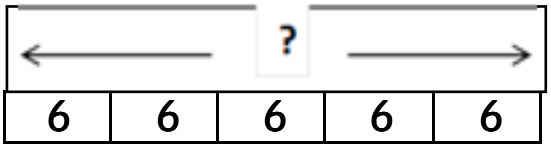
352 - 168 =

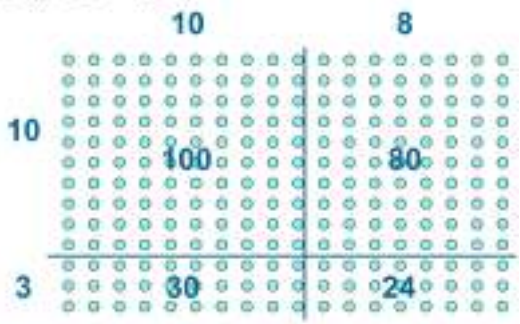
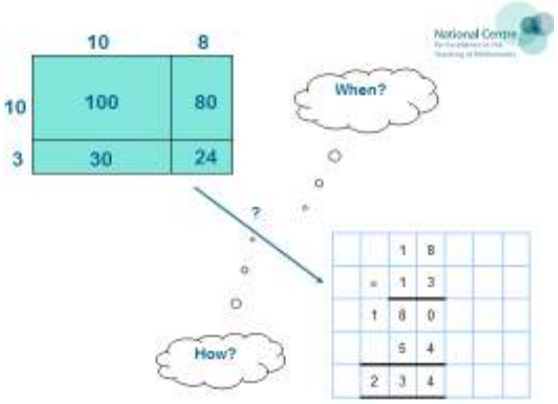
$$\begin{array}{r} 352 \\ -\ 168 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ 3\not{5}^{1}2 \\ -\ 168 \\ \hline 4 \end{array} \qquad \begin{array}{r} 2^{1}4 \\ \not{3}\not{5}^{1}2 \\ -\ 168 \\ \hline 184 \end{array} \qquad \begin{array}{r} 2^{1}4 \\ \not{3}\not{5}^{1}2 \\ -\ 168 \\ \hline 184 \end{array}$$

Stages of working shown

Written methods of calculation for Multiplication.

Step 1	
Concrete Multiplication *The first step requires the children to use objects or images to count in steps* *Language should be extended to: ____ lots of _____* *Use of fingers is encouraged as this is a constantly available resource.* *When discussing problems, this should also be introduced as a* **bar model**, *initially drawn round the objects, then blank.*	*For example:* 3 x 2 = 6  5 x 3 = 15 There are 3 sweets in one bag. How many sweets are there in 5 bags?  5 x 2 = 10   

Step 2	
Arrays *This step requires the children to use objects or pictures in arrays.* *It should be extended to arrays of dots or circles.*	For example: 2 x 3 = 6 3 x 2 = 6  3 x 4 = 12 4 x 3 = 12  3 x 6 = 18 6 x 3 = 18 
Step 3	
Repeated addition on number line *This step requires the children to show repeated addition using number lines.* *Counting in steps of __ should be practised at this point. Fingers can be used to keep track of how many lots of __ you have counted.* *Problem solving should include the use of* **bar modelling** *to aid visualisation.*	For example: 4 x 3 = 12  Counting in 5's: "5, 10, 15, 20"  *Problem solving:* *Pencils come in packs of 6. If I buy 5 packs, how many pencils do I have?* 

Step 4	
Partitioning *This step requires the children to use arrays to visualise the partitioning of larger numbers, then they can find the total of each section.* *This directly relates to the following step (Grid method).*	13 x 18 = 234  100 + 80 + 30 + 24 = 234 
Step 5	
Grid Method *This step requires the children to place their partitioned numbers into a grid. They multiply each part of the number before adding the partial results.* *This method should be used for multiplication by one and two digit numbers and can be extended to include decimals.*	For example: 123 x 5 = 615 | x | 100 | 20 | 3 | | 5 | 500 | 100 | 15 | **500 + 100 + 15 = 615** **or** 500 + 100 + 15 615 56 x 2.3 = 128.8 | x | 50 | 6 | | 2 | 100 | 12 | | 0.3 | 15 | 1.8 | 100.0 12.0 15.0 1.8 128.8 815 x 34 = | x | 800 | 10 | 5 | | 30 | 24000 | 300 | 150 | | 4 | 3200 | 40 | 20 | 24000 300 150 3200 40 20 27710
Step 6	

Extended column method

This step requires the children to set the calculation out on in column and then multiply each partition together ***(units, then tens, then hundreds)*** *before adding the partial calculation together.*

This method should be extended to multiplication by two and three digit numbers, and multiplication of decimals.

Children should describe what they do by referring to the actual values of the digits in the columns. For example, in 38 × 7 is 'thirty multiplied by seven', or 'three tens times 7 units', not 'three times seven', although the relationship to 3 × 7 should be stressed.

For example:

38 x 7 = 266

HTU

38

X 7

56

210

266

32

x 24

8 (4 x 2)

120 (4 x 30)

40 (20 x 2)

600 (20 x 30)

768

286 x 29 = 8294

THTU

286

x 29

54

720

1800

120

1600

4000

8294

Step 7

Short Method for x U

This step requires the children to use ***carrying*** *to shorten the method. This method can be used effectively for multiplication of decimals.*
The carried number should be placed ***underneath*** *the appropriate column*

For example:

38 x 7 = 266

HTU

38

X 7

266

934 x 6 = 5604

Th H T U

9 3 4

6

5 6 0 4

2 2

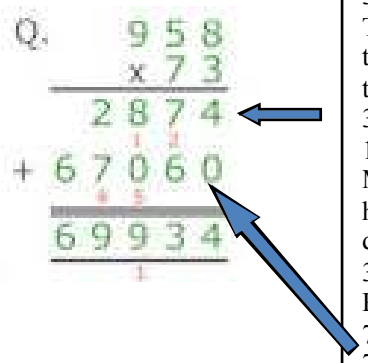
237 x 4 = 948

2 3 7

x 4

9 4 8

1 2

Step 8	
Short Method for x TU This method requires the children to multiply the larger number by the units and then the larger number by the tens, and so on, before adding the two numbers together. Carried numbers should once again be placed underneath the appropriate column..	For example:  Multiply the units first and carry any tens across: 3 x 8 = 24 (carry the 2 tens) Then multiply the units by the tens, add the carried digit, then carry again: 3 x 5 = 15 + 2 = 17 (carry the 1) Multiply the units by the hundreds, add the carried digit: 3 x 9 = 27 + 1 = 28 Repeat for the tens digit: 70 x 8 70 x 50 70 x 900

Written methods of calculation for Division.

Step 1

Concrete Sharing

The first step requires the children to use objects or images to share.
Language should be extended to: ____ shared by _____
The division number sentence should be shown alongside the calculations.

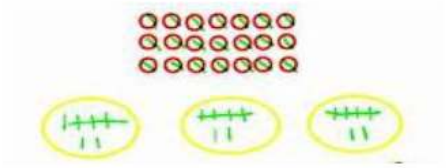
Remainders are expressed as 1 left, 2 left etc.

When discussing problems, this should also be introduced as a **bar model**, *initially drawn round the objects, then blank.*

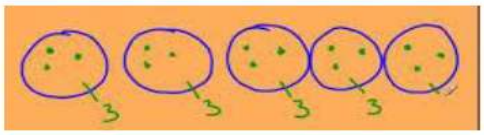
For example:

6 ÷ 2 = 3 (six shared by 2)

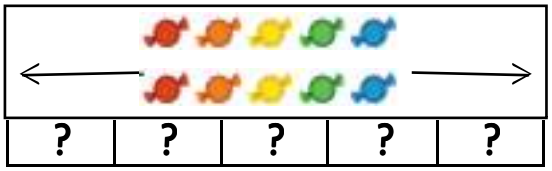
21 ÷ 3 = 7

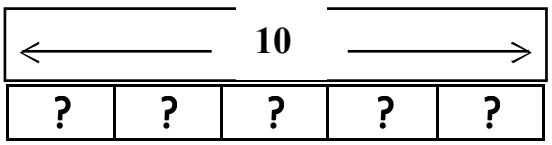


15 ÷ 5 = 3



I have 10 sweets, if I share them between 5 people, how many do they get each?





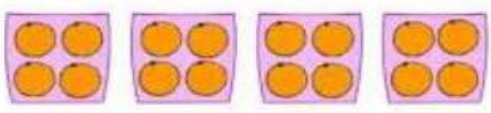
Step 2

Concrete Grouping

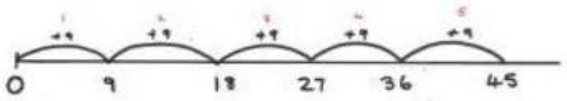
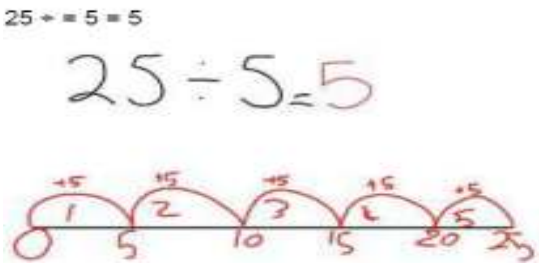
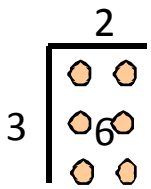
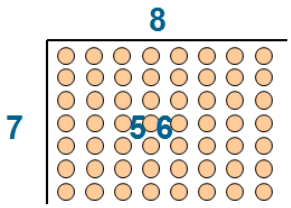
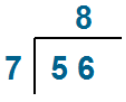
The children should recognise division as grouping as well as sharing. This can be done with objects or images.
Language should be extended to : How many groups of _____ can we get out of _____?
The division number sentence should be shown alongside the calculations.
Remainders are expressed as 1 left, 2 left There is also a need in some problems which leave a remainder to round up or down. Eg. How many bags do I need / how many full bags have I got, etc

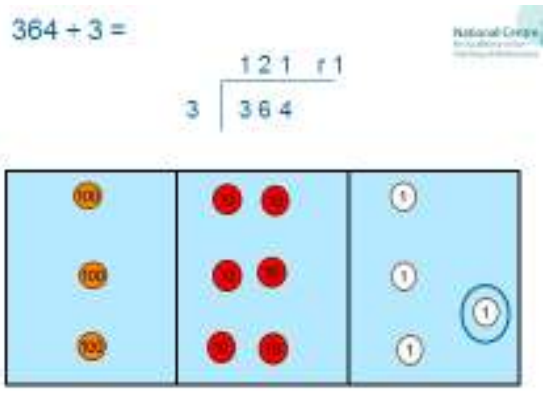
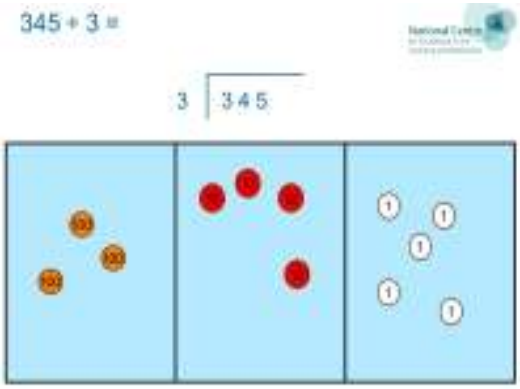
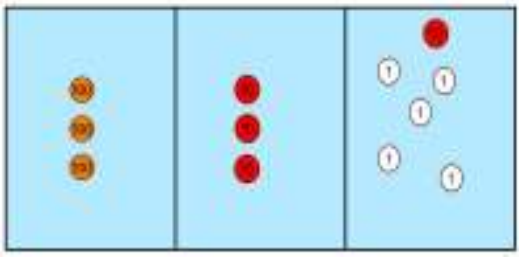
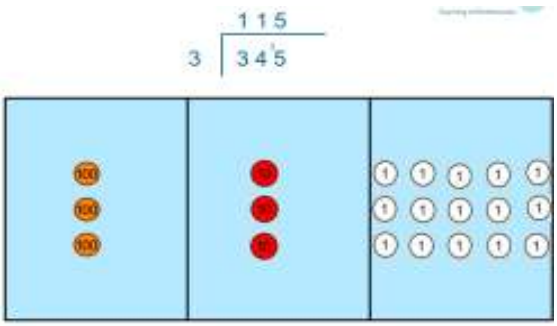
For example:
12 ÷ 4 = 3 (How many groups of 4 are there in 12?

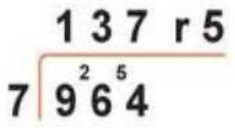
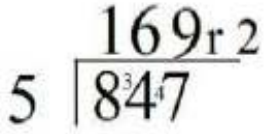
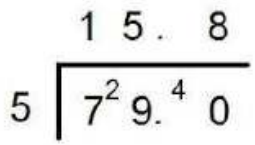
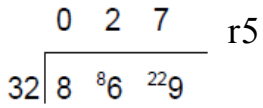
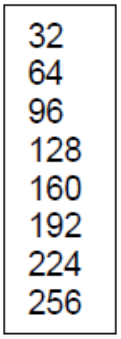
16 ÷ 4 = 4



When discussing problems, this should also be introduced as a **bar model** *as above.*	

Step 3	
Grouping on a number line *This step requires the children to count along a number line in relevant groups.* *This step should be extended to not using the number line and using tables knowledge or counting strategies.* *Remainders are referred to as remainders, but should be rounded up or down if appropriate to the problem.* *Counters should be used to group/share if needed.*	For example: 45 ÷ 9 = 5  
Step 4	
Use of arrays for division *This step requires the children to have the opportunity to explore how division relates inversely to multiplication. Use of arrays can highlight the links.* *Initially, they can identify the number family associated with the array.* *They can then be introduced to the more formal way of writing a division sum at this stage 'bus stop method'*	 2 x 3 = 6 3 x 2 = 6 6 ÷ 3 = 2 6 ÷ 2 = 3 The array is an image for division too  

Step 5	
Place Value Counters *This step requires the children to divide larger value digits using place value counters. The method should be used to divide TU and HTU by U* *Begin by dividing the* ***largest digit first*** *(eg hundreds, tens, then units)* *The dialogue going on in the learner's head, e.g. for 364 ÷ 3 I would ask myself 'How many groups of 3-hundreds can I make from the hundreds; how many groups of 3-tens can I make from the tens, etc* *The method should be extended to questions that require carrying.* *The method should be used* ***alongside step 6*** *to ensuring understanding of both.* *Remainders should be rounded up or down if appropriate to the problem.*	*For example:*  <u>Questions requiring carrying/exchanging:</u>  *Here, we need to carry / exchange one of the tens for 10 ones:*  

Step 6	
Short method for ÷ U ('Bus stop method') *This step requires the children to* **carry** *remainders within the calculation to make it more efficient. It should be used to divide TU, HTU, ThHTU as well as decimals.* *The method should initially be taught* **alongside step 5** *so the children understand what they are carrying and why.* *Decimal places should be added to show remainders as decimals.* *When problem solving, remainders should be rounded up or down if appropriate. Children can also be taught how to express remainders as fractions.*	*For example:* 964 ÷ 7 = 137 r5 or 137 5/7  847 ÷ 5 = 169r2 or 169 2/5  79 ÷ 5 = 15.8 
Step 7	
Short method for ÷ TU *This step requires the children to divide by TU. It requires the same method as step 5 although the children should be encouraged to write the tables of the divisor.*	*For example:* 869 ÷ 32 = 27 r5   32 64 96 128 160 192 224 256

Alternative method for carrying (long division)

Step 3: A remainder in the tens

In this step, students practice for the first time all the basic steps of long division algorithm: divide, multiply & subtract, drop down the next digit. We use two-digit numbers to keep it simple. Multiply & subtract has to do with finding the *remainder*, and after finding a remainder, we combine that with the next unit we are getting ready to divide (dropping down the digit).

An example:

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8	t o 2 2) 5 8 - 4 1	t o 2 9 2) 5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Step 4: A remainder in any of the place values

After the previous step has been mastered, students then practice long division with three- and four-digit numbers where they will have to go through the basic steps several times.

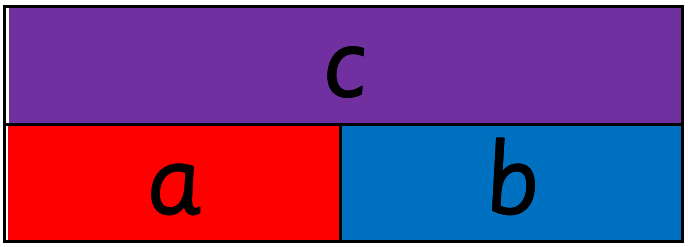
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2) 2 7 8	h t o 1 2) 2 7 8 - 2 0	h t o 1 8 2) 2 7 8 - 2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2) 2 7 8 - 2 0 7	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1	h t o 1 3 2) 2 7 8 - 2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0	h t o 1 3 9 2) 2 7 8 - 2 0 7 - 6 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

Notes for all calculations:

It is vitally important that children begin to use known facts and derive new facts from these as soon as possible.

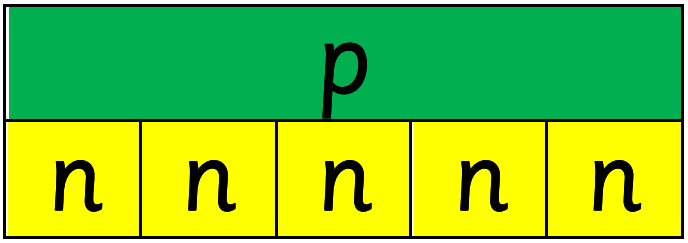
Eg.
If I know that 2 + 3 = 5, then I also know that 3 + 2 = 5.
From this I can work out that 5 – 3 = 2 and 5 – 2 = 3.
I can also see that 20 + 30 = 50, 30 + 20 = 50, 50 – 30 = 20 & 50 – 20 = 30

Children need to understand the relationship between operations.
Eg. subtraction is the opposite of addition, and division is the opposite of multiplication.



This bar model shows the relationship between addition and subtraction.

a + b = c
b + a = c
c – a = b
c – b = a



This bar model shows the relationship between multiplication and division.

5 x n = p
p ÷ 5 = n